Autonomous Drifting with 3 Minutes of Data via Learned Tire Models

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Beyond the limits of stability

The central problem

Why autonomous drifting?

- Assist drivers to regain vehicle control when facing the unexpected

Why is it challenging?

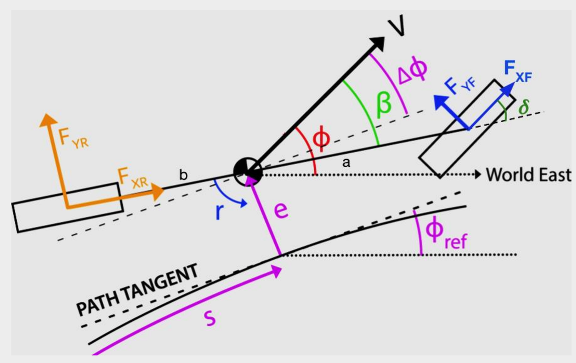
- Modeling tire forces becomes theoretically and empirically hard
  - Nonlinearities
  - Need special facilities to fit tires only, not the full interaction
  - Can we learn tire models directly from driving data?
- How to control?

Yaw rate, Velocity, Sideslip, Wheel speed, Path distance, Lateral error, Angular deviation

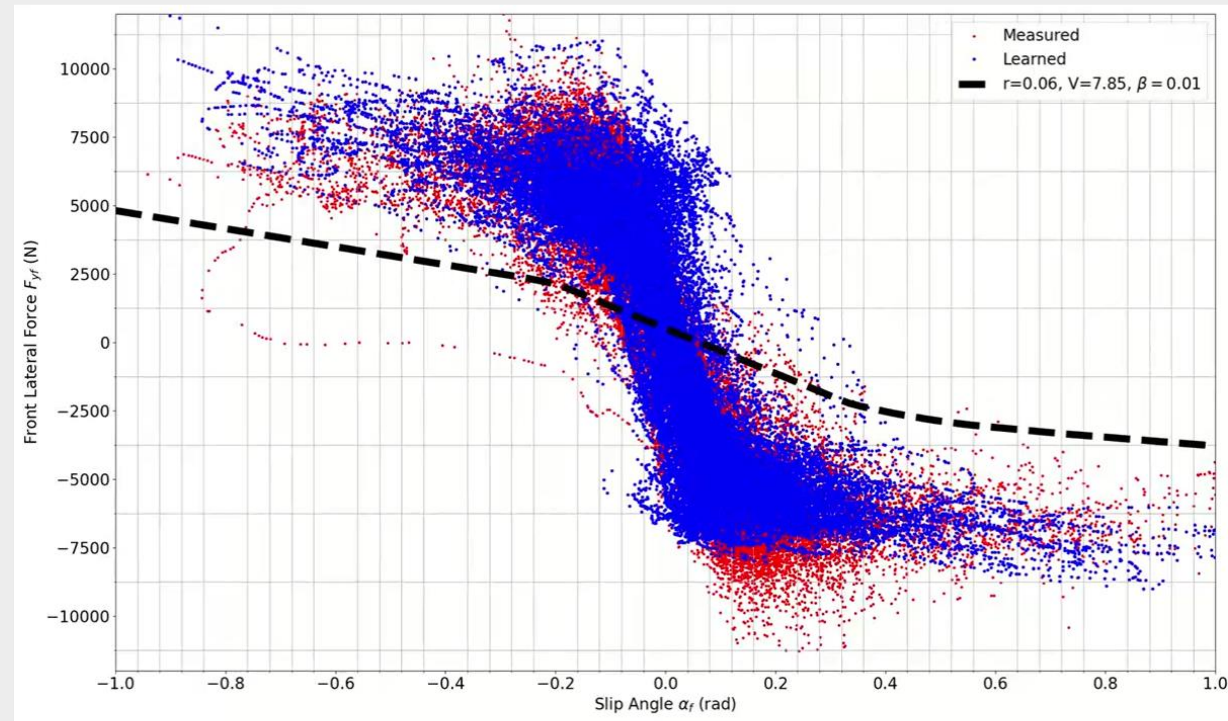
$x = [r, V, \beta, \omega_{f,r}, s, e, \Delta\Phi]$

$u = [\delta \coloneqq steering, \tau \coloneqq engine]$

$\dot{x} = M(x,u) \begin{bmatrix} F_{xf}(x,u) \\ F_{yf}(x,u) \\ F_{xr}(x,u) \\ F_{yr}(x,u) \end{bmatrix}$



Black box tire forces models do not generalize



Red points: Rough estimate of tire forces at states in the driving dataset

$\begin{bmatrix} F_{xf}(x,u) \\ F_{yf}(x,u) \\ F_{xr}(x,u) \\ F_{yr}(x,u) \end{bmatrix} = "(M(x,u))^{-1} \dot{x}"$

Blue points: Learned force model at a subset of points in the dataset

Black curve: Randomly pick r, V, β in dataset and evaluate learned force by sweeping slip angle

We need to incorporate some physics knowledge in the learning!

What do we know from tire fundamentals?

Informally speaking, we want the learned models to satisfy:

- S-shape characteristic
- Curve inflection points
- Friction limits: The maximum force is limited by a given (or learned) peak force $\overline{\mu F_z} = \bar{\mu} m g$, where $\bar{\mu}$ encodes any available knowledge on the road friction coefficient

Slip angle: $\alpha = \frac{V \sin \beta \pm dr}{V \cos \beta}$

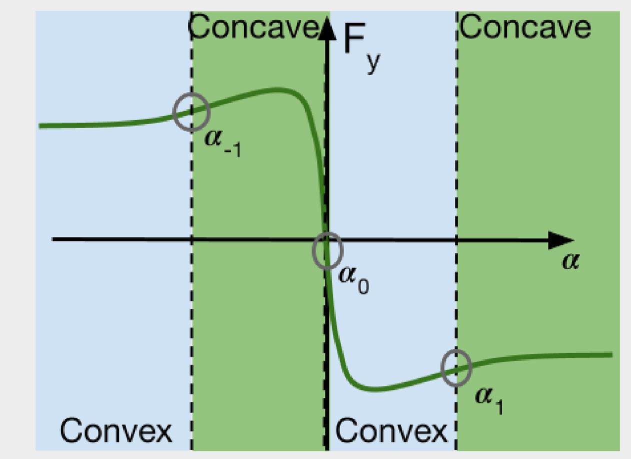
Slip ratio: $\sigma = \frac{r_\omega \omega - V}{V}$

Total slip: $\kappa = \sqrt{(\tan \alpha)^2 + \sigma^2}$

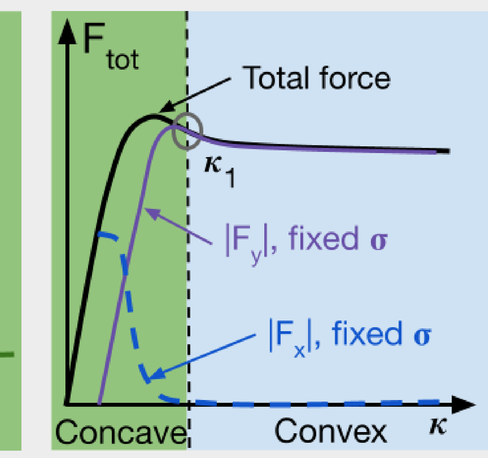
Total force: $F_{tot} = \sqrt{(F_x)^2 + (F_y)^2}$

d: COG to wheel axis distance

r_ω: Wheel radius



Pure slip regime ($\alpha = 0$ or $\sigma = 0$)



Combined slip regime ($\alpha \neq 0$ and $\sigma \neq 0$)

Solution approach

Our deep tire models with physics knowledge

Neural ODE-based tire models

Key idea: Write the force dynamics as a slip-varying second order neural ODE

Neural ExpTanh: A new family of tire models

Key idea: Intuit a function that satisfies the conditions on the second order ODE dynamics

e.g., in the *combined slip regime*, compute α, σ, κ from vehicle state. Then,

Neural ODE:

$\text{feat} = [r, V, \beta, \mu \bar{F}_z]$ — 1. Define networks inputs

$[\kappa_1^\theta, F_0^\theta, G_0^\theta] = \text{NN}_3^\theta(\text{feat})$ — 2. Inflection point and initial conditions of ODE

$[s_1^\theta, s_2^\theta] = \text{NN}_4^\theta(\alpha, \sigma)$ — 3. Distribution of total force

(1) $\dot{F}_{tot}^\theta = G_{tot}^\theta, \; z = [\kappa, F_{tot}^\theta, G_{tot}^\theta, \kappa_1^\theta]$

$\dot{G}_{tot}^\theta = \begin{cases} -\exp\{\text{NN}_1^\theta(z, \text{feat})\} & \text{if } \kappa \leq \kappa_1^\theta \\ \exp\{\text{NN}_2^\theta(z, \text{feat})\} & \text{otherwise} \end{cases}$ — 4. Total force dynamics. The derivative is wrt slip

$F_{tot}^\theta, - = \text{ode}((1), [\kappa_0^\theta, \kappa], [F_0^\theta, G_0^\theta])$ — 5. Total force via integration

$F_y^\theta = \frac{s_1^\theta F_{tot}^\theta}{\sqrt{(s_1^\theta)^2 + (s_2^\theta)^2}}, \; F_x^\theta = \frac{s_2^\theta F_{tot}^\theta}{\sqrt{(s_1^\theta)^2 + (s_2^\theta)^2}}$ — 6. Longitudinal and lateral forces

Neural ExpTanh:

$\text{feat} = [r, V, \beta, \mu \bar{F}_z]$

$(a_i^\theta)_{i=0}^5 = \text{NN}^\theta(\text{feat})$ — 2. Parameters of ExpTanh

$[s_1^\theta, s_2^\theta] = \text{NN}_4^\theta(\alpha, \sigma)$

$\text{ExpTanh}^\theta(z) = a_0^\theta + (a_1^\theta + a_2^\theta e^{-a_3^\theta |z|}) \tanh(a_4^\theta (z - a_5^\theta))$

$F_{tot}^\theta(\kappa, \text{feat}) = \text{ExpTanh}^\theta(\kappa)$

$F_y^\theta = \frac{s_1^\theta F_{tot}^\theta}{\sqrt{(s_1^\theta)^2 + (s_2^\theta)^2}}, \; F_x^\theta = \frac{s_2^\theta F_{tot}^\theta}{\sqrt{(s_1^\theta)^2 + (s_2^\theta)^2}}$

Training optimization problem:

$$\min_\theta \frac{1}{N} \sum_{\substack{\alpha,\sigma,r,V,\beta,\bar{F}_y, \\ \bar{F}_x, \mu \bar{F}_z \in \mathcal{D}}} \left(F_{tot}^\theta - \bar{F}_{tot}\right)^2 + (F_y^\theta - \bar{F}_y)^2 + (F_x^\theta - \bar{F}_x)^2$$

Friction limits soft constraint:

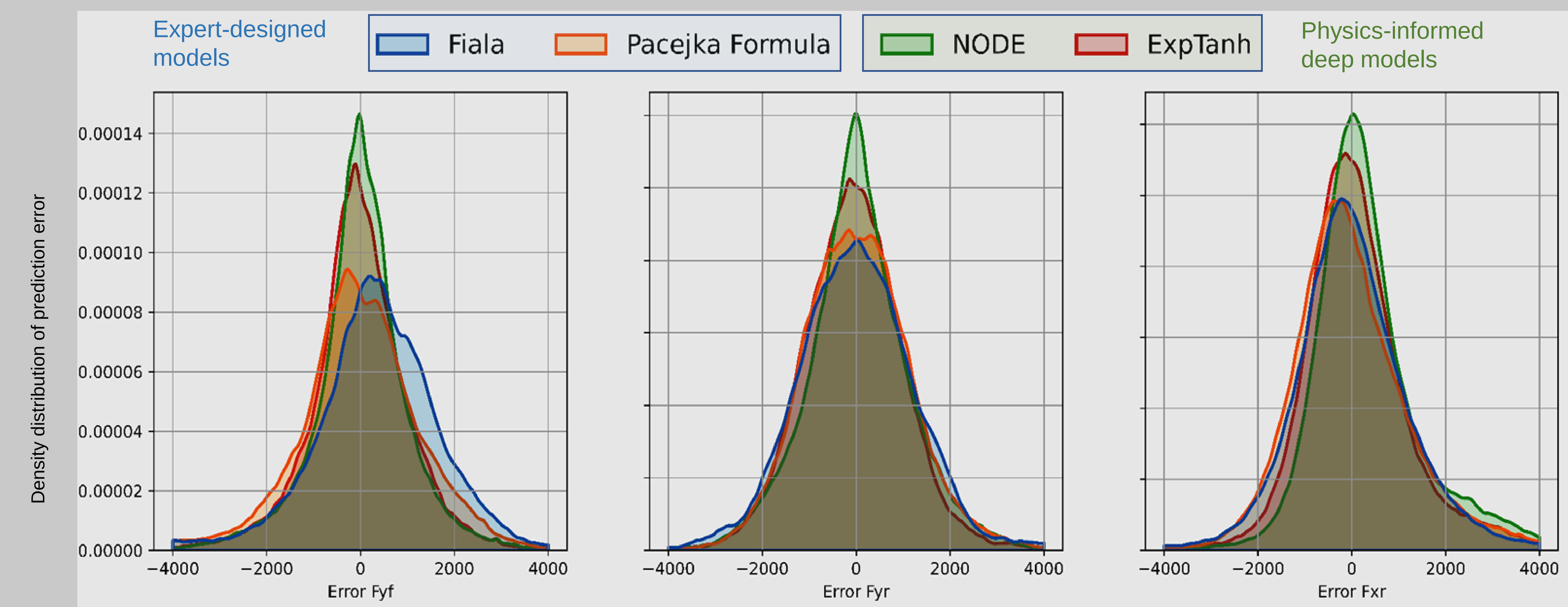
$+ \lambda(\min\{\mu \bar{F}_z - F_{tot}^\theta, 0\})^2$ — Neural ODE

$+ \lambda(\mu \bar{F}_z - F_{tot}^\theta(\kappa_+^\theta, \text{feat}))^2$ — ExpTanh

Analytic expression for the maximum/minimum curve values of ExpTanh:

$$z_\pm^\theta = a_5^\theta \pm \text{atanh}\left(\frac{\sqrt{(a_2^\theta a_3^\theta)^2 + 4(a_4^\theta)^2} - a_2^\theta a_3^\theta}{2 a_4^\theta}\right)$$

Comparison with existing baselines



Our models are $\geq 1.5 \times$ denser around zero-mean error than the baselines on a real-world driving dataset

Drifting experiments

Learning to autonomously drift with a Toyota Supra

Keisuke: GPS, IMU, computer-controlled steering, throttle, clutch, braking, …

3 minutes worth of driving data

Scenario: Nonlinear model predictive control (MPC) for autonomous drift using the learned tire models

- Manually collect 3 minutes of data
- Learn tire models
- Return on the track to drift

$$\underset{u_i, x_i}{\text{minimize}} \sum_{i=1}^{N-1} c(x_i, u_i) + c_{\text{end}}(x_N)$$

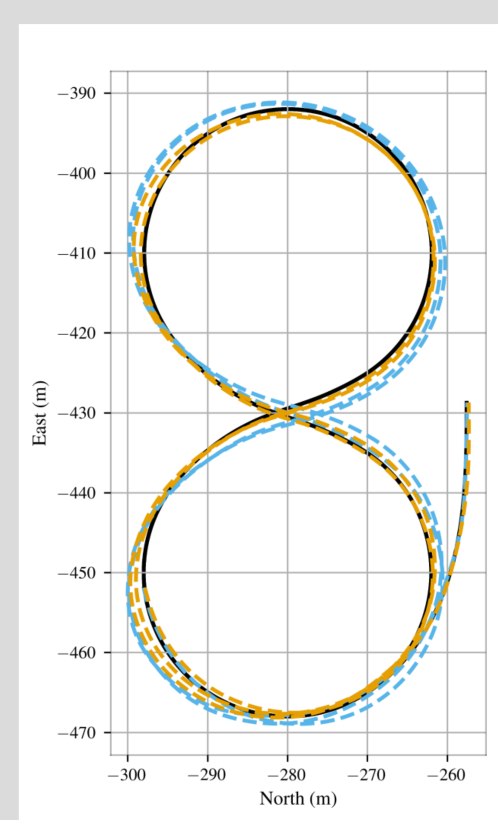
$$\text{subject to} \quad x_{i+1} = \text{ode}(M(\cdot)[F_{xf}, F_{yf}, F_{xr}, F_{yr}], t_i, x_i]) \quad ((\omega_r)_i, \delta_i, \tau_i) \in \mathcal{U}$$

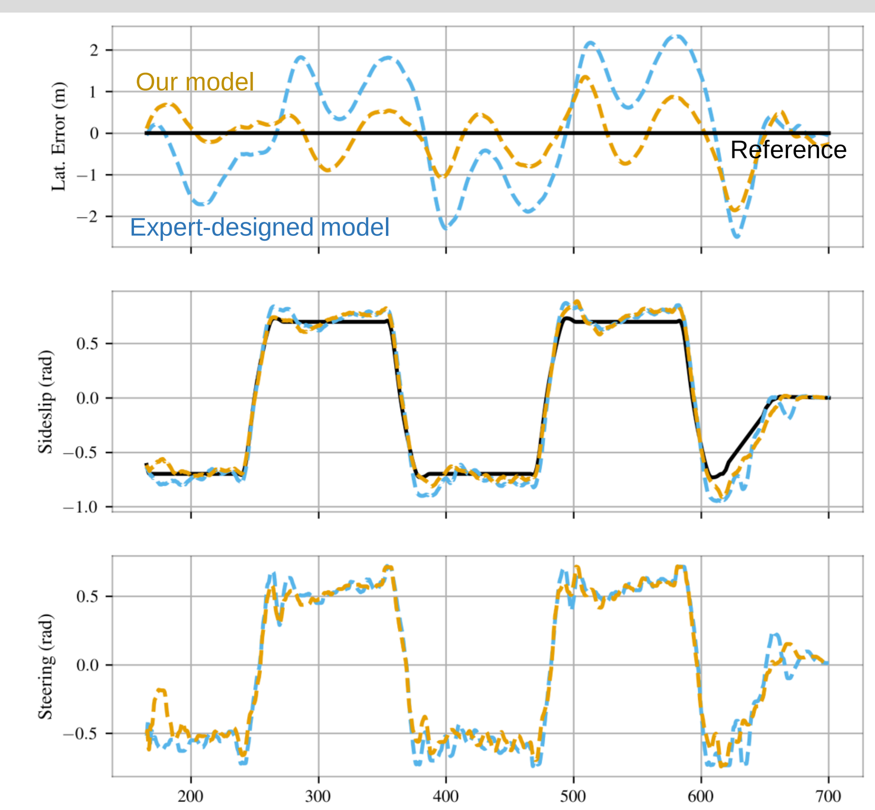
$$c(x_i, u_i) = e_i^2 + 20\Delta\phi_i^2 + 60(\beta_i - \beta^{\text{ref}}(s_i))^2 + 0.5((\omega_r)_i - \omega_r^{\text{ref}}(s_i))^2 + (\dot{\delta}_i)^2 + 10^{-8}(\dot{\tau}_i)^2$$

$$c_{\text{end}}(x) = 10(\dot{e} + e)^2 + 10((\dot{\beta} - \dot{\beta}^{\text{ref}}(s_N))^2 + 2(\beta - \beta^{\text{ref}}(s_N)))^2$$

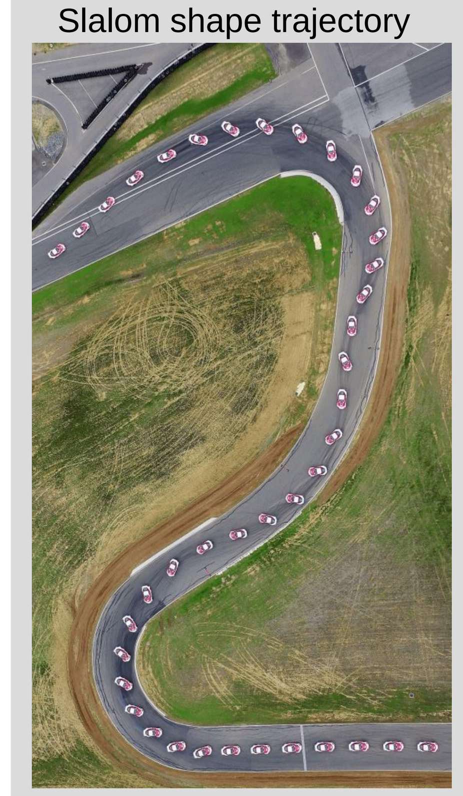
MPC: Track a pre-computed trajectory while following a sideslip profile

Figure-8 shape trajectory





Slalom shape trajectory



Source: Toyota Research Institute