On-The-Fly, Data-driven Reachability Analysis and Control of Unknown Systems: An F-16 Aircraft Case Study

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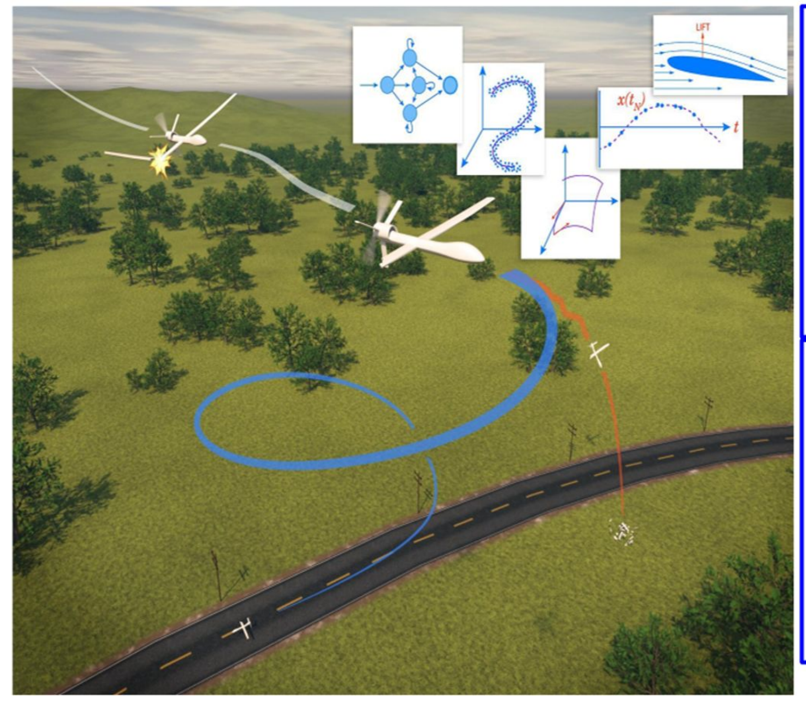
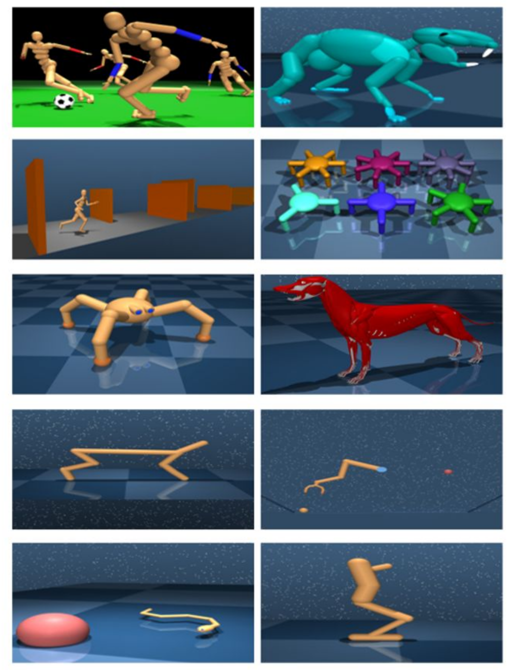
Learning On The Fly from Severely Limited Data

Aircraft undergoes unexpected damages during flight



Source: The Aviationist

Learning for robotics



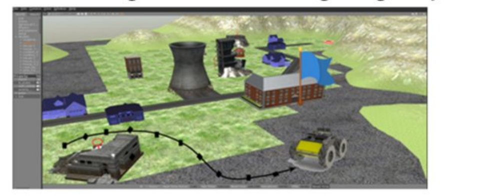
Source: DeepMind

Is it even possible to learn how to control in one attempt?

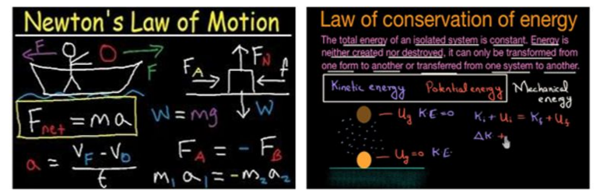
Learning to Control in One Attempt? No Problem

Given:

- Streaming data from ongoing trajectory



- Side information, if available

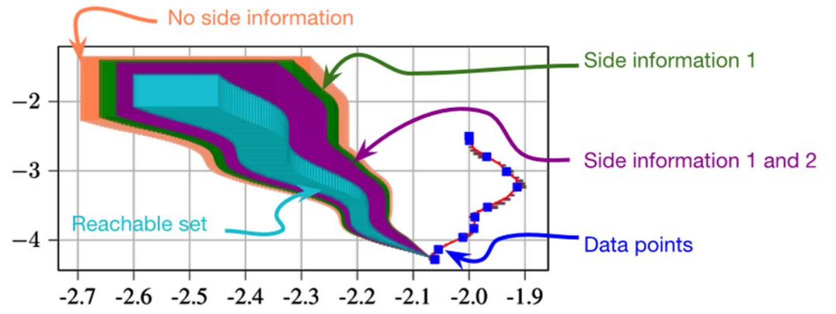


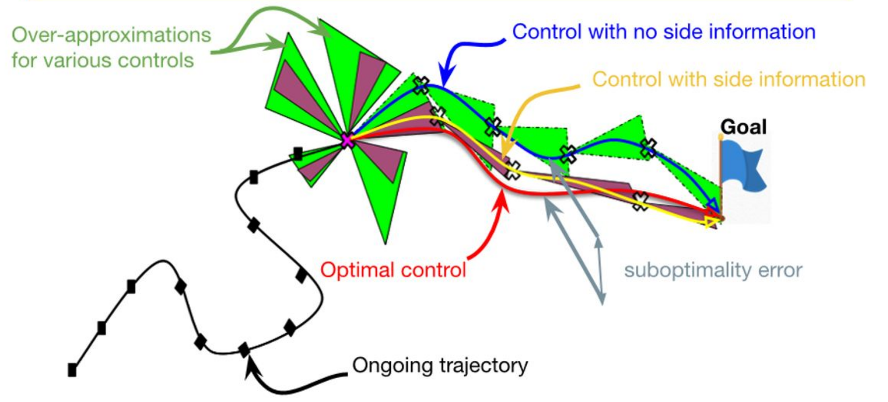
Data-driven over-approximations of reachable sets

- Crucial for safety concerns
- Tighter sets with more data or side information

On-the-fly control with provable guarantees

- Near optimality: Bound on suboptimality error
- Real-time: Bound on number of flops[1]





[1] F. Djeumou et al, "On-The-Fly Control of Unknown Systems: From Side Information to Performance Guarantees through Reachability", IEEE-TAC, 2020

Formal Problem Description

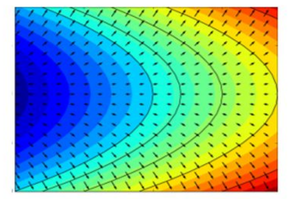
Consider a system with **unknown nonlinear** dynamics in control-affine form

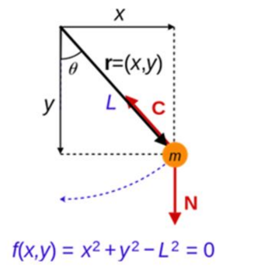
$$\dot{x} = f(x) + G(x)u$$

- u: Constrained signal of time in $\mathcal{U} \subseteq \mathbb{R}^m$
- f, G: Unknown Lipschitz functions on $\mathcal{X} \subseteq \mathbb{R}^n$

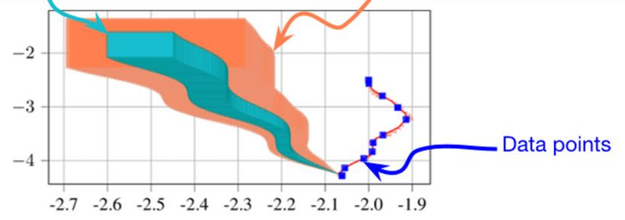
Given:

- Data from **ongoing trajectory** $\{x(t_i), \dot{x}(t_i), u(t_i)\}_{i=1}^{N}$
- **Side information**
  - Lipschitz bounds
  - Local gradient bounds
  - Algebraic constraints
  - Known terms of dynamics





Problem 1: Compute an over-approximation of the reachable set of the system



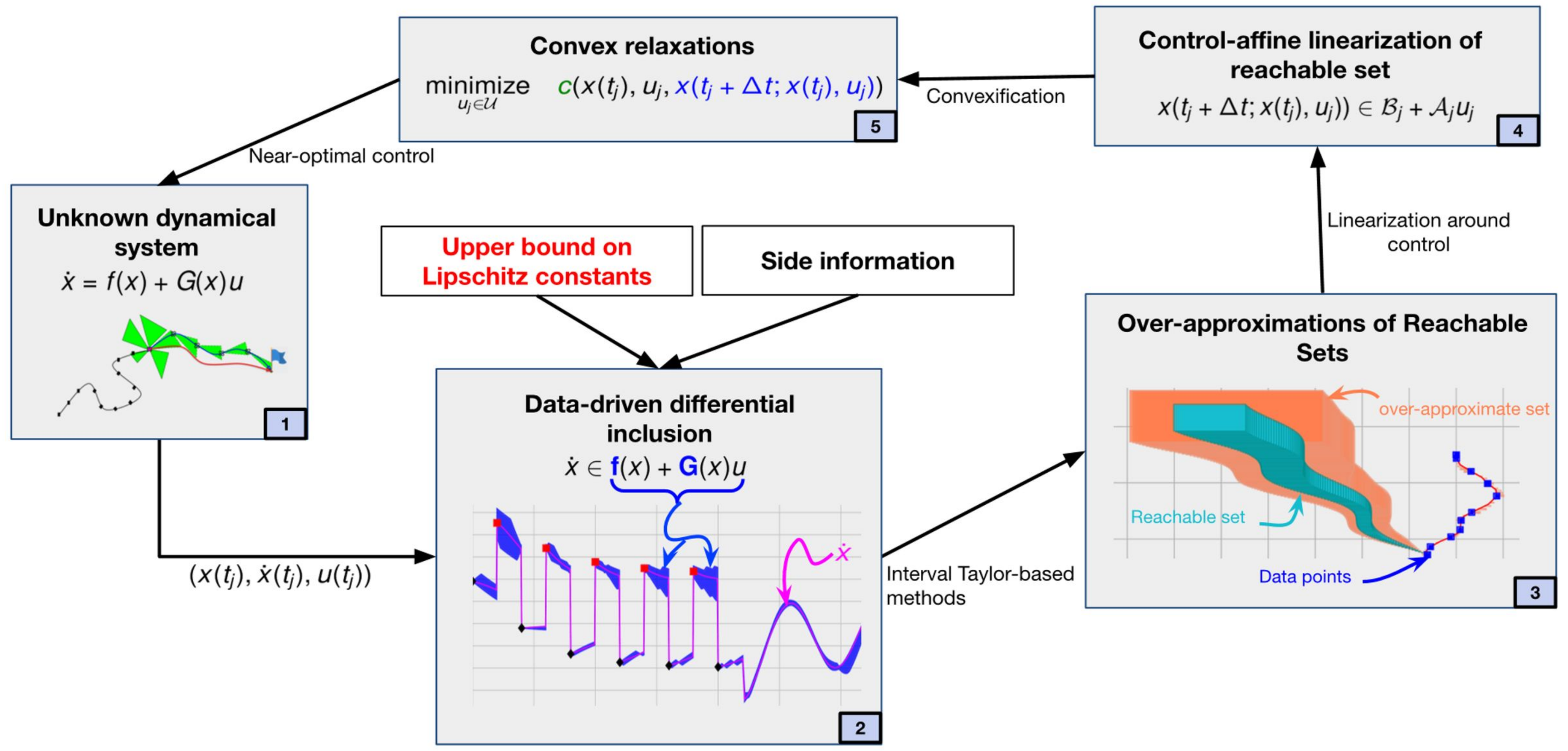
Problem 2: At each time t_j, Compute a solution to the **one-step optimal control** problem

$$\underset{u_j \in \mathcal{U}}{\text{minimize}} \quad c(x(t_j), u_j, x(t_j + \Delta t; x(t_j), u_j))$$

Cost function c; Unknown future state $x(t_j + \Delta t; x(t_j), u_j)$

We can only hope for approximate solutions

Solution Approach in a Nutshell



Data-Driven Over-Approximations of Reachable Sets

Closed-form expression for over-approximations of reachable sets

$$\mathcal{R}^{+}_{j+1} = \mathcal{R}^{+}_j + \Big(f(\mathcal{R}^{+}_j) + G(\mathcal{R}^{+}_j)\mathcal{V}\Big)\Delta t + G(\mathcal{S}_j)\mathcal{V}^{(1)}\frac{\Delta t^2}{2} + \Big(\frac{\partial f}{\partial x}(\mathcal{S}_j) + \frac{\partial G}{\partial x}(\mathcal{S}_j)\mathcal{V}\Big)\Big(f(\mathcal{S}_j) + G(\mathcal{S}_j)\mathcal{V}\Big)\frac{\Delta t^2}{2}$$

Known dynamics case (Over-approximation; Current set of states; Set of control values; Control derivatives; A Rough enclosure)

$$\mathcal{R}^{+}_{j+1} = \mathcal{R}^{+}_j + \Big(\mathbf{f}(\mathcal{R}^{+}_j) + \mathbf{G}(\mathcal{R}^{+}_j)\mathcal{V}\Big)\Delta t + \mathbf{G}(\mathcal{S}_j)\mathcal{V}^{(1)}\frac{\Delta t^2}{2} + \Big(\mathcal{J}_f + \mathcal{J}_G\mathcal{V}\Big)\Big(\mathbf{f}(\mathcal{S}_j) + \mathbf{G}(\mathcal{S}_j)\mathcal{V}\Big)\frac{\Delta t^2}{2}$$

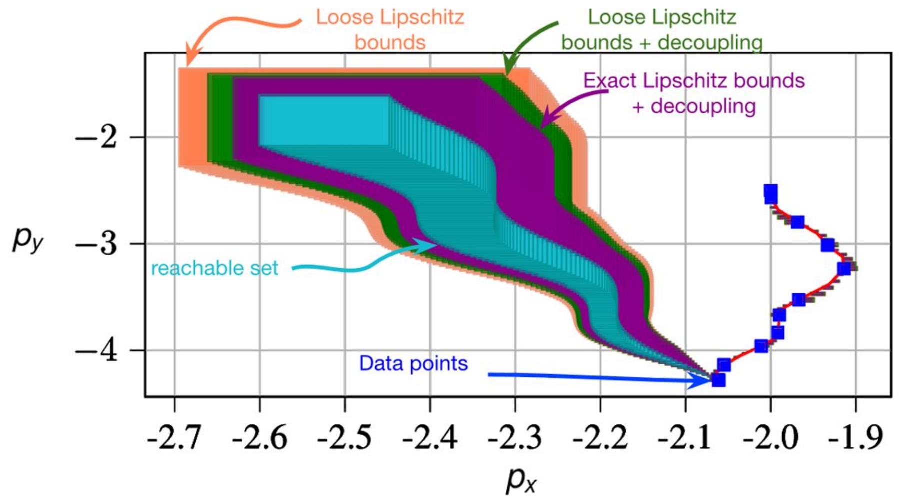
Unknown dynamics case (Over-approximations of Jacobian values)

Side information provides tighter sets

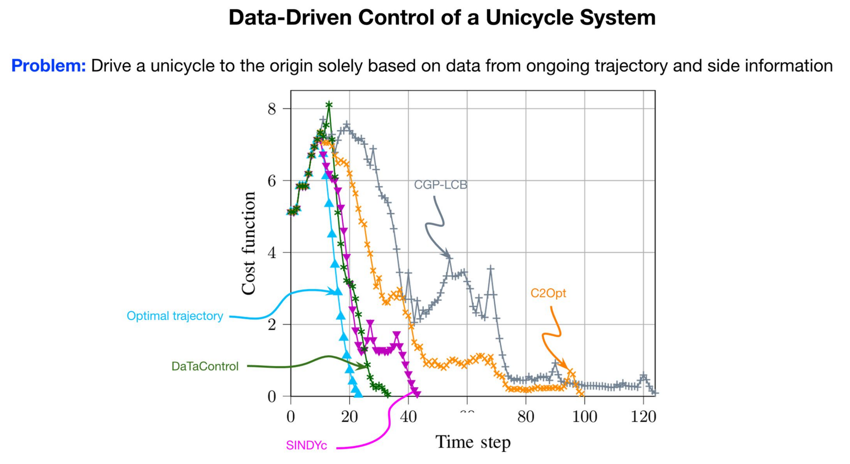
Example: A unicycle system

$$\dot{p}_x = v\cos(\theta),\ \dot{p}_y = v\sin(\theta),\ \dot{\theta} = \omega$$

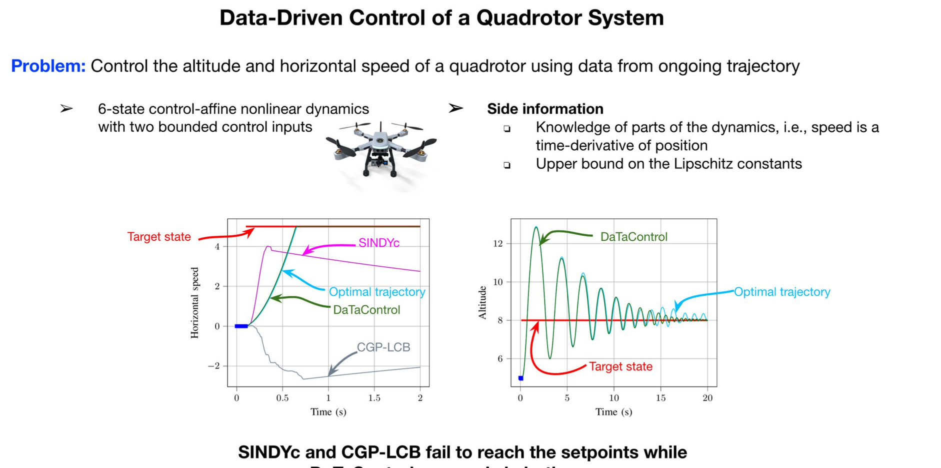
- Loose Lipschitz bounds
- Decoupling: Vector field independent of positions



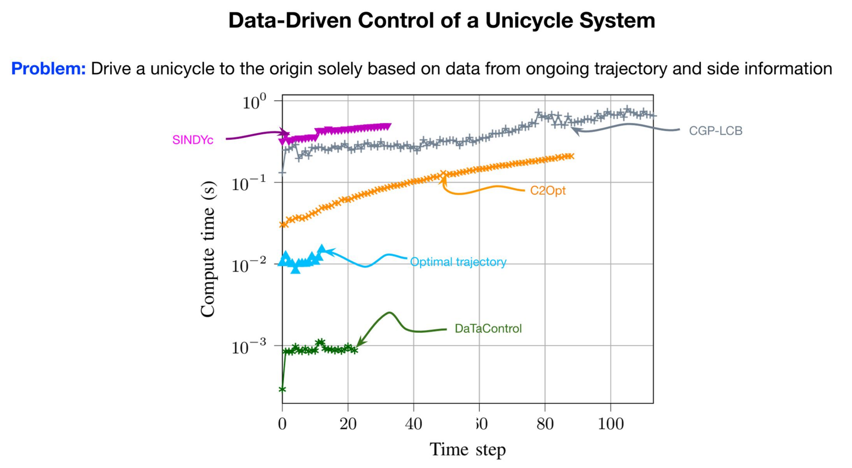
Results



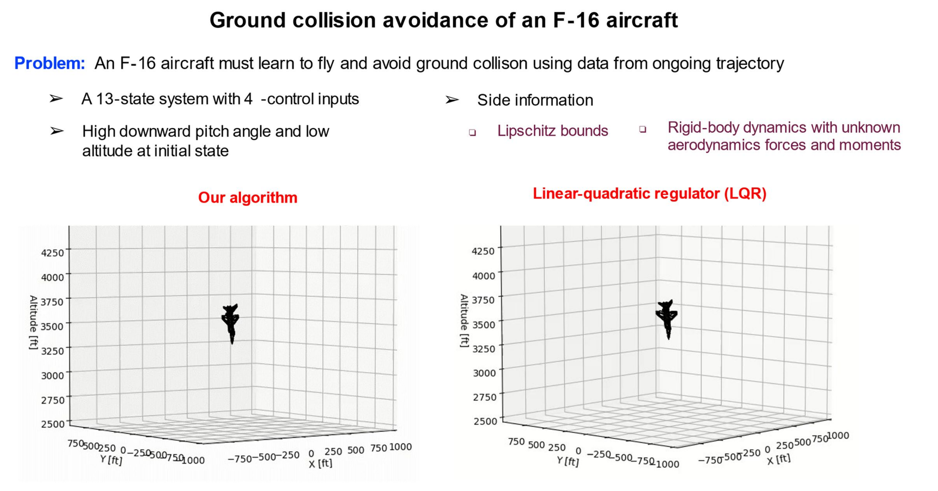
DaTaControl performs significantly better than existing approaches



SINDYc and CGP-LCB fail to reach the setpoints while DaTaControl succeeds in both cases



DaTaControl is computationally faster than existing approaches



Our algorithm avoids ground collision while the tuned LQR of the simulator fails

Additional Content:
A video for the F-16 aircraft experiment is available at https://tinyurl.com/4hhynbba

References:
1. F. Djeumou et al, On-The-Fly Control of Unknown Systems: From Side Information to Performance Guarantees through Reachability, IEEE-TAC (in review), 2020
2. F. Djeumou et al, On-The-Fly Control of Unknown Smooth Systems from Limited Data, American Control Conference (ACC), 2021

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